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A SOFTWARE FOR KINETOSTATIC SYNTHESIS OF COMPLIANT MECHANISMS

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ABSTRACT

This paper presents a computer program for kinetostatic synthesis for design automation of compliant mechanisms. Kinetostatic synthesis is solving the geometric (e.g. link lengths) and elastic parameters (e.g. spring constants) for a prescribed set of kinematic and static force specifications. Although many kinematic synthesis algorithms and methods for compliant mechanism synthesis are available, a unified software tool that integrates algorithms and methods is yet to be developed. In our previous work, we have developed a unified framework for kinematic and static analysis of rigid body and compliant mechanisms. In this work, we extend this framework to kinetostatic synthesis of compliant mechanisms. Optimization algorithms for kinetostatic synthesis problems are presented and examples from different kinetostatic synthesis modules such as the bistable and constant force compliant mechanisms are given to demonstrate the current capability.

1 INTRODUCTION

Rigid-body [1] mechanisms transform motion, force, or energy with rigid links joined by movable joints. Instead of using movable joints, same tasks and some mobility can be obtained from the the deflection of flexible members and this type of mechanisms are called complaint mechanisms [2]. Kinetic analysis needs to be performed on a mechanism to understand if displacements, velocities, and accelerations are satisfactory for required function of the mechanism. Also kinetic analysis is the backbone of the kinematic synthesis to design mechanisms for achieving desired motion. For compliant mechanisms, static analysis should accompany kinematic analysis to determine the relation between design loads and resulting mechanism motion. Later this relation can be used in kinetostatic synthesis of compliant mechanisms for obtaining the specified relation between the input and output forces and the deflections.

A number of academic and commercial softwares are available for kinematic synthesis of mechanism. However most softwares are limited to a specific type of mechanism. Sphinx [3] is limited to mechanisms with four revolute joints whereas Spades [4] can only used for synthesis of mechanisms with four cylindrical joints. 4-Bar, 5-bar and 6-bar linkage synthesis can be performed with Linkages [5]. Only spatial linkages with serial chain synthesis can be carried out with Synthetica [6]. Kinematic synthesis tools for RRR and RPR legged parallel manipulators [7] and for mechanisms with six revolute joints [8] were developed. SAM [9] is only software that is capable of kinematic synthesis of any designed mechanism via optimization.

Even though methods [10] for compliant mechanism synthesis has been developed for quite a long time, currently there is no software available for unified design and synthesis of compliant mechanisms. Synthesis by topology optimization (structural optimization) [11–13] and kinetostatic synthesis [14] are the two major methods in compliant mechanism synthesis. Topology optimization is based on minimizing total strain energy or total potential energy of whole structure using finite element methods and it is more suited to truss type mechanisms. Kinetostatic synthesis employs pseudo-rigid-body models (PRB Models) [15] for

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representing elastic members of compliant mechanism to obtain corresponding rigid-body mechanism and then use loop closure equations and virtual work to find the relation between loads and deformation.

Previously a kinetostatic framework was developed [16] which is a design and analysis tool for planar rigid or compliant mechanisms. Currently, the framework is capable of kinematic and static analysis of rigid-body and compliant mechanisms and GUI for the framework is under development. Ultimate goal is adding kinematic synthesis of rigid-body mechanisms and kinetostatic synthesis of compliant mechanisms. In this paper, preliminary kinetostatic synthesis capability of the framework is investigated.

2 KINETOSTATIC SYNTHESIS OF COMPLIANT MECHANISMS

Kinetostatic synthesis is performed to determine geometric dimensions and elastic parameters for specified design requirements including kinematic motion and static force equilibrium constraints.

The basis of the static analysis was developed in [16]. After converting the compliant links into a series of rigid-body links connected with torsional or linear springs, the static analysis was performed by minimizing total potential energy of the system:

$$\min_{\psi_i}\left(\sum_{i=0}^{n}\frac{1}{2}K_\theta\psi_i^2-\sum_{k=0}^{n}\int_{r_{k0}}^{r_k}F.\mathrm{d}r-\sum_{l=0}^{m}\int_{\theta_{l0}}^{\theta_l}M.\mathrm{d}\theta\right) \tag{1}$$

where the first summation is the energy stored in linear or torsional springs and the two integrals are the work done by forces and moments, respectively.

A special case of Eq.(1) will occur if one of the rigid links is being driven by a unknown moment M or a slider (horizontal or vertical) is driven by a unknown force in the same direction. For these three cases, total potential energy becomes:

$$\min_{\psi_i}\left(\sum_{i=1}^{n}\frac{1}{2}K_\theta\psi_i^2-F\triangle\right) \tag{2}$$

where for the rigid link, horizontal slider and vertical slider $F\triangle$ is equal to $M_z\delta\theta$, $F_x\delta x$ and $F_y\delta y$, respectively.

After the link or the slider moves by δ amount, the corresponding total potential energy equation must be minimized to determine the remaining link angles or slider positions. Although the acting moment or force is unknown, the terms ($M.\delta\theta$, $F_x.\delta x$ and $F_y.\delta y$) on the right side of the energy equations are constant and it will not make any difference if they are omitted. The objective function is reduced to:

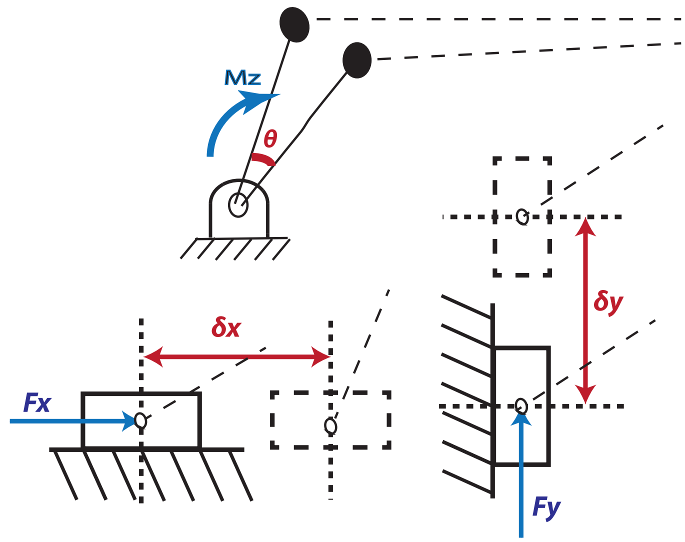

Figure 1 : Three cases are shown where simplified equation can be used.

$$\min_{\psi_i}\left(\sum_{i=1}^{n}\frac{1}{2}K_\theta\psi_i^2\right) \tag{3}$$

Eq.(3) is nothing but the total energy stored in springs and when this energy is minimized subjected to the kinematic equation constraint, the final configuration of the mechanism can be obtained even if the mechanism has multi-degrees of freedom. Total work done on the mechanism by the loads $W=\int F\mathrm{d}\triangle$ is stored in the springs and equal to the total potential energy (Eq.(3)). The relation between loads and total potential energy can be found by taking derivative of total potential energy:

$$F=\frac{\mathrm{d}U}{\mathrm{d}\triangle} \tag{4}$$

where for the rigid link, horizontal slider and vertical slider $(F,\triangle)$ is equal to $(M_z,\delta\theta)$, $(F_x,\delta x)$ and $(F_y,\delta y)$, respectively.

Eqs.(3) and (4) are very useful in kinetostatic synthesis of single load compliant mechanisms because of the simplicity compared to Eq.(1). These equations are easier to use and it was found that these equations greatly increase convergence of the problem and decrease computation time in finding the total energy and the driving load. Algorithm 1 shows the procedure for finding energy and torque values within a defined range. The analyzed mechanism is designed using the framework before calling the algorithm and then algorithm starts with creating an equally spaced displacement list between initial and target displacement which can be a link rotation or a slider displacement.

At each iteration towards the desired displacement, the corresponding link angle or slider position is assigned and optimizer is called to minimize Eq.(3) subjected to kinematic constraints of the mechanism. After finding the unknown angles or the displacements of the components of the rest of the mechanism, total potential energy is calculated and torque is calculated by taking derivative of the potential energy. The derivative is approximated using a central finite difference method. Finally, if the calculated torque is zero, the stability of the current point is investigated and the current point is reported as a instable or a stable point.

Algorithm 1 Algorithm for obtaining torque and energy within a specified range

```
procedure CALCULATE TORQUE AND ENERGY
    distance=[0,...,100]_n
    for i ← 1 to n do
        Link Angle(Slider Position)←angle(distance)[i]
        function MINIMIZE ENERGY
            minimize Eq.(3)              ▷ Subject to kinematic equations
            return the configuration
        calculate Energy(i)              ▷ Eq. (3)
        Torque(i)← derivative of Energy(i)
```

Fig. 2 shows an actual output of the software resulting from Algorithm 1. A partially compliant four-bar mechanism was designed using the framework and Algorithm 1 is called using the design as the input. For this example, running time of the algorithm is 15 seconds without real time plotting and 45 seconds with real time plotting.

2.1 Synthesis for prescribed flexural stiffness

A general case of compliant mechanism synthesis arises when loads acting on the mechanism are known and a specific deflection is desired. In this case, the role of the module is determining the cross-section (I) and material (E) properties of the flexible members in the mechanism. In this case, overall design of the mechanism is known but the flexural stiffnesses (EI) of the compliant members are unknown. Therefore, the number of unknowns are equal to the number of compliant members.

An overview of this module is shown in the Algorithm 2. The desired mechanism is designed, loads acting on the mechanism are defined and a desired deflection is set using the framework. The desired deflection can be either motion of a node or angular deflection of a rigid link. During a optimization step, first flexural stiffnesses of the compliant members which will be used as initial guesses are assigned. The compliant members are converted to a series of rigidly connected links using the PRB Model which was predefined by the user at the design stage. With the mechanism fully defined, the static solver is called and current displacement is calculated. Optimization is terminated when the desired displacement is obtained. Overall design (lengths of the members) is known before the optimization, so a good initial guess can be supplied to the optimizer and usually convergence is obtained very easily. However if one optimization fails to reach the target, the difference between the current and target position is divided into a number of steps and incremental optimizations are performed.

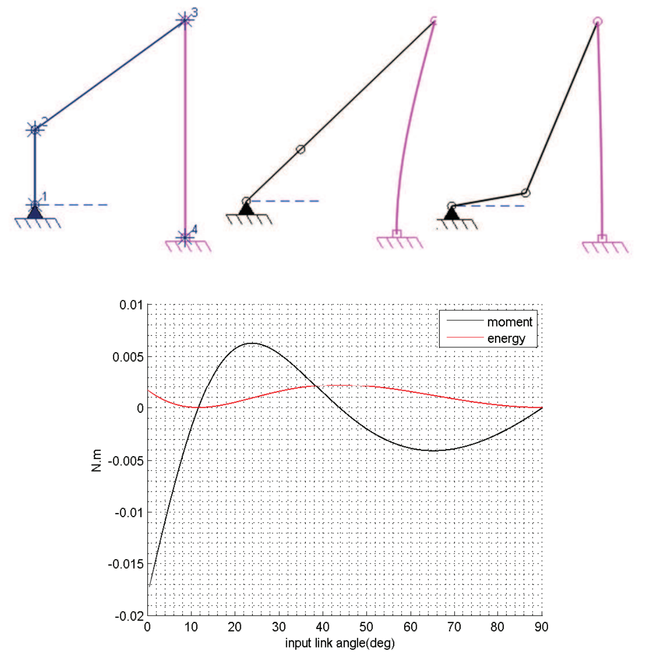

Figure 2 : Partially compliant link shown at three equilibrium points. Also driving torque and energy graph are given. Compliant link is divided into 10 equal rigid-body segments.

Algorithm 2 Algorithm for the flexural stiffness synthesis of compliant mechanisms

```
Design the mechanism                       ▷ with the framework
Define the loads                           ▷ with the framework
Define the target distance                 ▷ with the framework
function OPTIMIZATION(Variables:EI_i)
    Set EI_i for compliant members
    Convert compliant members with PRB Models
    Perform force analysis                 ▷ with the framework
    Compare the current and target displacement
```

2.2 Synthesis of Bistable Mechanism Synthesis for Specified Critical Load

Bistable mechanisms are the mechanisms that have two equilibrium points. The mechanism will stay at these points without requiring any external force. Compliant bistable mechanisms [17] acquire two stable states by storing energy at their compliant segments (Fig.3). Critical load is defined as the maximum external load (force or moment) that is required to actuate the mechanism from one stable to the other stable positions.

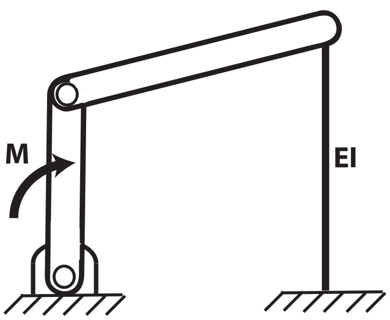

Figure 3 : A partially compliant bistable mechanism is shown.

Fig. 4 shows the flowchart for the bistable mechanism synthesis module. First a compliant mechanism is designed using the framework and a target distance for the driven link or slider is defined. Variation of the total potential energy and the torque between initial and final configuration are calculated before the synthesis procedure to verify instable and stable points exist in the defined range. First synthesis mode (Optimization Variables:EI_i) is for finding the required stiffness(es) for the compliant beam(s) that will result in specified critical load. Alternatively, in the second synthesis (Optimization Variables:l_i) desired instable and stable positions of the mechanism can be determined and the link lengths can be found via optimization. At each optimization step, Algorithm 1 is called to find current instable and stable points or critical torque. Since instable and stable point already exist between the design and target positions, the global convergence of the optimization is obtained at almost hundred percent of the cases and because of this high level convergence the optimization is always performed once.

2.3 Synthesis of Constant Force Compliant Mechanisms

Constant force mechanisms are the mechanisms that maintain a constant output force during a deflection range. Compliant members or compliant joints can also be used in a constant force mechanism [2]. Fig. 5 shows a compliant constant force mechanism with three flexible joints. These flexible joints are approximated as pin joints with torsion springs. The link length

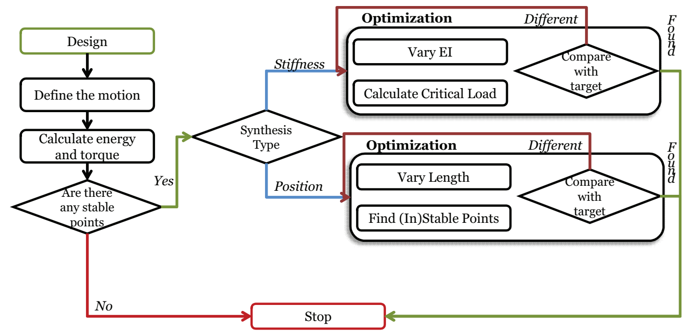

Figure 4 : The flowchart for the two bistable synthesis problems is shown.

and torsional spring ratios must be established in a way that will result in constant output force at the slider.

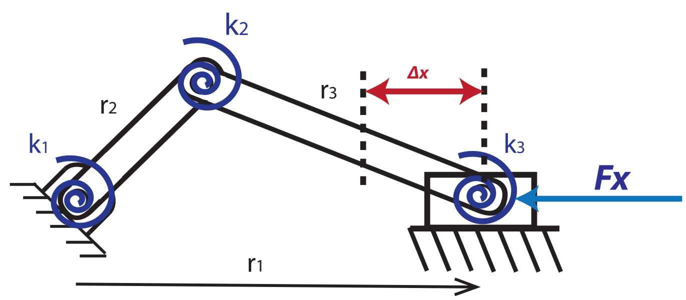

Figure 5 : An example constant force mechanism is shown.

Algorithm 3 shows the overall procedure for the constant force mechanism synthesis module. The framework is employed in designing the mechanism and in defining the expected total distance of constant force. Some of the link lengths, spring values or flexural stiffness of compliant members can be given a constant value. Apart from these constant members, a link, spring and a compliant member will be selected as the reference and other member values will be a multiple of the this reference. In other synthesis problems, a target was set and the role of the optimization was achieving this predefined goal. However no target value exists in this problem and a global minimum of the ratio Fmax and Fmin is sought. Usually compared to other synthesis problems more variables exist and they are more diverse which increases the probability of being stuck at a local minimum. Therefore the main optimization procedure is repeated at least 10 times with random variables. For each kind of member (link,spring and compliant) range of random variables are different. During the optimization the ratios can result in a mechanism for which the resulting motion is less than the specified value so no force ratio can be calculated. If such a case is detected, the main optimization function returns NaN (Not a number). It was observed that a optimizer with approximated derivatives is much successful in overcoming NaN values and converging near a minimum. Therefore two different optimizers (first derivative based, then derivative free) are called at each optimization step. During an optimization step, first the mechanism is constructed using the current ratios and then energy and torque variation are calculated for a range which is double of the desired motion. Finally the minimum and maximum force ratio is calculated in intervals equal to the desired motion. After doing the optimization procedure n times, the best configuration is reported to the user.

Algorithm 3 Algorithm for constant force mechanism synthesis

```
Design the mechanism                              ▷ with the framework
Define the target distance                        ▷ with the framework
Select the references
for i ← 1 to n do
    Randomly assign initial conditions
    function OPTIMIZATION 1 AND 2
        Adjust link ratios
        Adjust joint spring rations
        Adjust EI ratios for compliant members
        Convert compliant members with PRB Models
        Calculate Energy Variation  ▷ of size m, using Alg. 1
        Calculate Load Variation    ▷ of size m, using Alg. 1
        ratio=DOUBLEMAX
        for j ← 1 to m/2 − 1 do
            FminCurrent=min(Loads(1:j))
            FmaxCurrent=max(Loads(1:j))
            currentRatio=FmaxCurrent/FminCurrent
            if currentRatio < ratio then
                ratio=currentRatio
```

3 CASE STUDIES

In this section, three case studies are presented to demonstrate the kinetostatic synthesis software described in the previous section.

3.1 Parallelogram Flexure Mechanism

A compliant parallelogram flexure is selected to test flexural stiffness synthesis module. The parallelogram consists (Fig. 6) of two compliant beams and one rigid on which the forces and the moment act. Links are same length (100 mm) and compliant beams are automatically approximated with 10 equal rigid-beam segments. 15 mm (in y direction) deflection of the third node is desired for the synthesis problem.

Table 1 and Fig. 6 show the results of the synthesis process. For the first case, the same flexural stiffness was assumed for the both flexible members and different flexural stiffnesses were assigned during optimization for the second case. The optimization took around two minutes to complete for both of the cases. The result for the first case can be verified via the BCM method [18] and the y displacement of the rigid link is found as 15.108 mm (0.72% difference) from the nonlinear BCM equation.

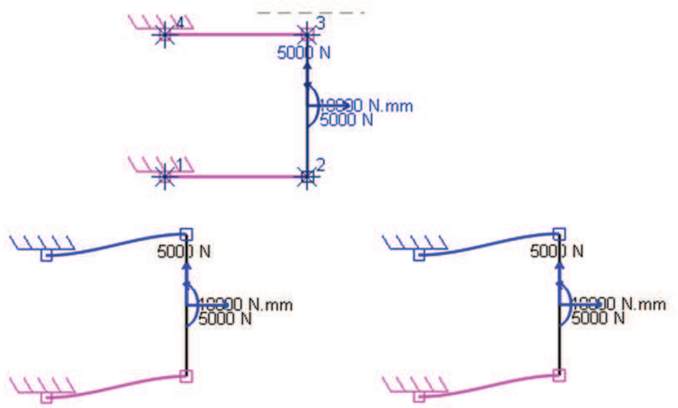

Figure 6 : At top the initial configuration of the parallelogram is shown from the output of the software. The synthesis result after assuming same flexural stiffness for both compliant link is shown at bottom left. At bottom right, the synthesis result is shown with different flexural stiffnesses for the compliant beams.

Table 1 : The synthesis optimization results are shown for the compliant links.

Case	$EI_1(N.m^2)$	$EI_2(N.m^2)$
1	10.9315	10.9315
2	13.2264	8.6324

3.2 Slider Crank Constant Force Mechanism

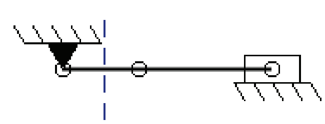

Figure 7 : The slider crank designed using the software is shown before synthesis optimization.

A compliant slider crank type mechanism (Fig. 5 and 7) was chosen to test constant force mechanism synthesis module. The objective of the synthesis is selected as having a constant output force at the slider through slider displacement of $0.4(r_2+r_3)$. The constant output force is not expected to begin at the initial configuration, therefore the slider was displaced for a distance of $0.8(r_2+r_3)$ and this total distance was swept with intervals of length $0.4(r_2+r_3)$ to find smallest Fmax/Fmin ratio.

Fig. 8 shows the some possible compliant variants of the rigid body mechanism shown at Fig. 7. In the rigid-body mechanism three pin joints exist and these pin joints can be replaced with flexible small length flexural pivots. Torsional springs at pin joints approximate the behavior of the small length segments. The ratio of link lengths and the ratios of the torsional spring magnitudes determine the constant force behavior of the mechanism at each case and they are the variables that are optimized at each case.

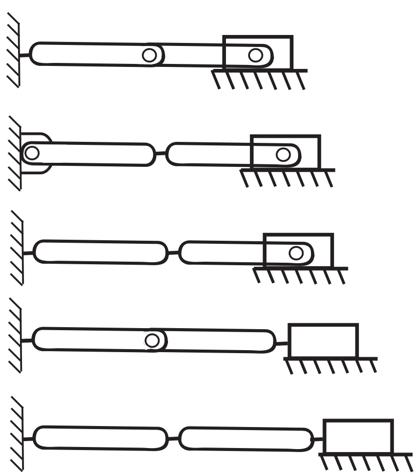

Figure 8 : Five synthesis cases for the constant force mechanism is shown.

Table 2 show the results of the constant force synthesis for the five cases. R, K_2 and K_3 denote the ratios $r_3/\ r_2$, $k_2/\ k_1$ and $k_3/\ k_1$, respectively. For the first two cases, optimization and reference values [2] completely agree at length(R) and force ratios. The optimizer was able to find a better value for the forth case and although the force ratios are similar in cases 3 and 5, the configuration (R,K_2 and K_3) is entirely different.

3.3 Four Bar Bistable Mechanism

A four bar mechanism was designed to test the bistable mechanism synthesis module and it is shown in Fig. 9 with the target rotation of 90deg which is known to contain instable and stable points. Compliant, crank and coupler lengths are 43.294 mm, 15 mm and 37.0842 mm, respectively. With initial 1.5 mm width, 5 mm depth and 1.38 GPa Young's Modulus, the critical torque is around 0.005 N.m and the objective is having a critical torque of 0.02 N.m. Since only flexural stiffness of the compliant link is optimized, the location of the instable and stable points will remain same.

Table 2 : The synthesis optimization results are shown for the five cases.

Case	R	K_2	K_3	$\frac{Fmax}{Fmin}$	$\frac{Fmax}{Fmin}$ [2]
1	0.8867	-	-	1.0243	1.0241
2	1.0	-	-	1.1579	1.1576
3	0.8644	0.0447	-	1.0093	1.0058
4	1.8584	-	4.7211	1.0046	1.1914
5	1.9134	0.1547	5.4497	1.0048	1.0049

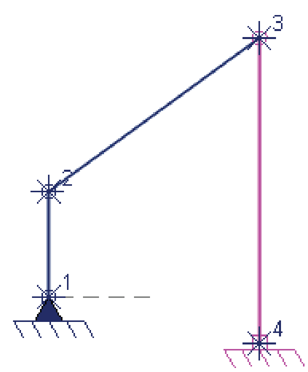

Figure 9 : The bistable mechanism with the target distance is shown.

The synthesis results with a EI value of 0.009564 $N.m^2$ in just two iterations. Fig. 10 shows the initial and the final energy-torque curves. It can be seen that after finding the instable point the algorithm only runs until the instable point since the torque values until the unstable point is important.

4 CONCLUSION AND FUTURE WORK

The past two decades has witnessed extensive research of compliant mechanisms with numerous sophisticated synthesis theories and algorithms developed. However, there is a lack

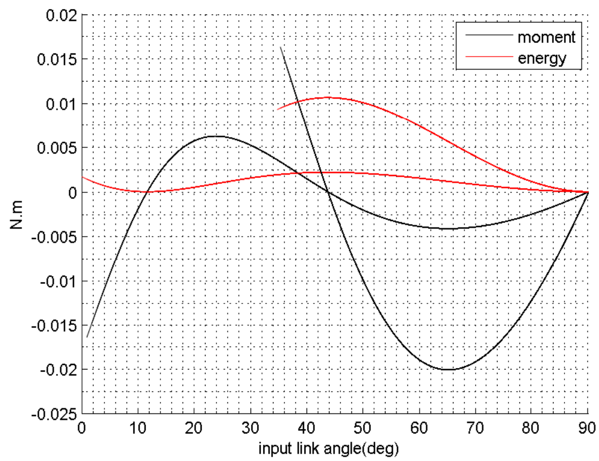

Figure 10 : The bistable mechanism with the target distance is shown.

of software tool that integrate these computational methods into a comprehensive design tool for compliant mechanisms. Previously, a unified kinetostatic framework had been developed which is capable of kinematic and static analysis of compliant mechanisms. The framework uses PRB methods to represent compliant members. The main objective of this paper is extending this framework to incorporate compliant mechanism synthesis tools.

First the main energy equation of the framework was simplified for a particular case that is common in compliant mechanism synthesis. Three synthesis modules were developed: synthesis for prescribed flexural stiffness, synthesis of bistable mechanisms, and synthesis of constant force mechanisms. The first synthesis module employs the unsimplified energy equation whereas the others use the simplified model. The underlying algorithm for each module was shown and an example for each module was presented in the last section. The synthesis targets were easily achieved with the bistable mechanism and prescribed flexural stiffness synthesis. Dissimilar results from the reference were found for the constant force mechanism synthesis. Even when the results were similar, the optimized configuration differed. This supports the argument of starting from random points and optimizing more than once to find global maximum for this synthesis problem.

Our ultimate goal is developed a unified synthesis tool for both kinematic synthesis of rigid body mechanisms and kinetostatic synthesis of compliant mechanisms. To achieve this goal, more synthesis modules needs to be added in the future. Also current modules will be upgraded to support multi-target optimization such as finding flexural stiffnesses which will also be smallest weight. Finally a GUI for the software needs to be developed to visually aid users during the synthesis process.

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